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Folding model and coupled-channels analysis of ^{6,7}Li elastic and inelastic scattering

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Abstract. The double-folding optical potentials of ^{6,7}Li nuclei have been constructed using the S1Y effective nucleon-nucleon interaction. Angular distributions of the differential cross-section of ^{6,7}Li elastic and inelastic scattering from ^{12}C, ^{28}Si and ^{58}Ni targets in the energy range 12–35 MeV/u were analyzed using the derived potentials through the coupled-channels mechanism. Successful descriptions of the data were obtained using the derived potentials.

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1 Introduction

Usually, the primary optical model analysis of the heavy-ion (HI) scattering is performed using phenomenological six (or more) parameters Woods-Saxon (WS) potentials. However, a satisfactory microscopic understanding of the scattering process can be achieved if one relates the optical potential to a fundamental nucleon-nucleon (NN) interaction through the double-folding (DF) approach by folding this interaction with the nuclear-matter distributions of both projectile and target nuclei [1,2]. During the last two decades, or so, the HI elastic-scattering data were successfully predicted using the DF potential as the real part of the optical potential, while the imaginary potential was fitted in a phenomenological form. However, an anomalous behavior was noticed [2,3] for ^{6,7}Li nuclei, where the folded potential must be reduced by a renormalization coefficient ($N_{\rm R}$ = 0.5-0.6) in order to reproduce the data. This anomaly was attributed to the effect of the breakup of the loosely bound ^{6,7}Li nuclei and the factor $N_{\rm R}$ could be completely accounted for by considering explicit coupling of the elastic channel with the low-lying breakup channels [3,4]. As the incident energy increases the breakup channel, coupling effect usually decreases and the value of $N_{\rm R}$ approaches 1.0.

In the folding model analysis the choice of the nucleon-nucleon (NN) interaction is very crucial. Recently [5,6], an intensive and extensive study was performed to investigate DF optical potentials for ^{6,7}Li projectiles. Four different effective NN interactions, M3Y [7], S1Y [8], KH [9] and JLM [10], were used to derive these potentials. Twenty-six sets of angular distributions of the differential cross-section of ^{6,7}Li elastic scattering from ^{12}C, ^{28}Si, ^{40}Ca, ^{58}Ni, ^{90}Zr and ^{208}Pb targets at 12.5–53 MeV/u were analyzed using the derived potentials. Successful predictions of the observed cross-sections were obtained by these potentials. The M3Y and S1Y interactions provided the best predictions of the data. The KH interaction came next in this respect. The JLM interaction revealed an impotent ability to describe some sets of the analyzed data, particularly at the backward angles. It was found [5,6] that the anomaly of ^{6,7}Li projectiles through the folding model procedure dominates overall these potentials. The M3Y and S1Y potentials showed similar behaviors for the required renormalization factors and energy dependence of the volume integrals.

It has long been advocated [11] that heavy-ion (HI) optical model potentials determined from elastic scattering should be tested by their ability to reproduce cross-sections for other reaction channels, such as inelastic channels, since the form factors may probe different regions of the nuclear potential. Studying inelastic-scattering data can also be used to determine collective properties of nuclei. Analyses of the first 2^+ state of even-even nuclei are expected to provide quadrupole deformation parameters (QDPs), β_2, which could be determined either from electromagnetic measurements [12], or using hadronic probes, where a deformed optical potential is used to describe the data.

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The analysis of inelastic-scattering data can be performed using the folding model in the framework of the distorted-wave Born approximation (DWBA) or coupled-channels (CC) mechanisms. In DWBA the distorted wave functions for the incoming and outgoing particles are generated using the optical potential, which describes elastic scattering and the effect of the non-elastic channels is taken into account through the imaginary part of the optical potential. If, however, one or more inelastic channels are strongly coupled to the elastic channel it is not sufficiently accurate to take them into account through the optical potential when calculating the elastic scattering, nor is it sufficiently accurate to use DWBA to calculate the scattering into these inelastic channels [13]. In the CC approximation, as in DWBA, there are essentially no free parameters except those which occur in the model chosen for the nuclear wave functions. The CC mechanism has the additional and very important feature that the wave functions for elastic and inelastic scattering are calculated simultaneously.

Both DWBA and CC calculations were carried out [14] for the data of 210 MeV ^{6}Li elastic and inelastic scattering from ^{12}C, ^{28}Si and ^{58}Ni, based on the collective-model description of the first-excited (2^+) states of the target nuclei. Unique ^{6}Li phenomenological optical model potentials were used in these calculations [14].

The present work is a complementary part of our previous study [5,6]. In this part our purpose is to validate the ability of the S1Y interaction to reproduce the inelastic-scattering data. In addition, we aim to investigate the effect of coupling the inelastic channels to the elastic channel on the elastic-scattering calculation. We chose eight sets of ^{6,7}Li inelastic-scattering data in the energy range 12–35 MeV/u from different targets to be analyzed using DF optical potentials based on the S1Y interaction in the framework of the CC method. There are three sets at 210 MeV [14] besides five other sets at lower energies: two sets for ^{6}Li + ^{12}C at 124 and 169 MeV [15], two sets for ^{6}Li + ^{58}Ni at 71 MeV [16] and the fifth one is for ^{7}Li + ^{12}C at 132 MeV [15]. We consider the inelastic scattering to the first-excited (2^+) states of the target nuclei. For ^{6}Li + ^{58}Ni at 71 MeV two excited states ($2^+, 4^+$) are taken into account. Although there is an importance of explicitly including the ^{7}Li ground-state quadrupole moment and the coupling to the first-excited state of ^{6}Li nucleus in the calculations [17], we ignored the effect of coupling to lithium ions excited states, where breakup effects may be absorbed by the renormalization of the derived DF potentials [3,4]. In sect. 2 the formalism of the folding model is described. The results of the calculations and discussion are presented in sect. 3 and general conclusions are given in sect. 4.

2 Formalism

Inelastic-scattering measurements are usually analyzed using a deformed optical model potential (DP) [18,1]. This provides a transition potential whose radial dependence is

$$U_\ell^{\rm DP}(r) = -\delta_\ell^U \frac{{\rm d}U(r)}{{\rm d}r}\,, \tag{1}$$

where the deformation length δ_ℓ^U determines the strength of the interaction and $U(r)$ is the complex optical potential determined by the measured elastic scattering. As noticed from eq. (1) the shape of $U_\ell^{\rm DP}$ is independent of the multipolarity ℓ of the transition. The DP model (1) is often justified by arguing that the potential $U(r)$ follows the shape of the density distribution when the latter is deformed. However, a more direct and consistent application of this view is to generate both optical and transition potentials by folding an effective NN interaction over the ground-state density of the projectile and the deformed density of the target [1]. The transition density may be obtained microscopically from the nuclear-structure calculation (such as one using the random-phase approximation) [1], or macroscopically by deforming the ground-state density distribution [19,20].

In the present work, we use the latter prescription which is the analogue of the DP (1). For the transition of the multipolarity ℓ from a ground state with spin $J_{\rm i}$ to a final state with spin $J_{\rm f}$, we obtain the transition density as

$$\rho_\ell^{\rm tr}(r) = -\delta_\ell^\rho \frac{{\rm d}\rho(r)}{{\rm d}r} \tag{2}$$

where $\rho(r)$ is the ground-state density distribution of the target nucleus, and δ_ℓ^ρ is its 2^ℓ-pole deformation length, which is determined by the normalization to the electromagnetic transition probability $B(EL)$ as

$$B(EL, J_{\rm i} \to J_{\rm f}) = \frac{e^2}{2J_{\rm i}+1}\left|\frac{Z}{A}\int_0^\infty \rho^{\rm tr}(r) r^{\ell+2}{\rm d}r\right|^2\,, \tag{3}$$

where A and Z are mass and atomic numbers of the excited nucleus, respectively. The considered ground-state densities of the projectile and target nuclei are the same as used in ref. [5].

The S1Y interaction has the form

$$v_{\rm nn}(s) = v_0 \frac{e^{(-s/0.7)}}{(s/0.7)}\,, \tag{4}$$

where v_0 is the real depth of the interaction. We use the same folding model procedure as described before [5,6]. However, in the present work we confine the folding procedure to the real part while the imaginary part is considered in a phenomenological Woods-Saxon (WS) form. The imaginary transition potential is obtained from the derivative of the imaginary central potential as

$$W_\ell^{\rm tr}(r) = -W_0 \delta_\ell^{\rm W} \frac{\rm d}{{\rm d}r}\left[1+\exp\frac{(r-r_{\rm I}A^{1/3})}{a_{\rm I}}\right]^{-1}\,. \tag{5}$$

The imaginary deformation length, $\delta_\ell^{\rm W}$, is defined as $\delta_\ell^{\rm W} = \beta_\ell^{\rm W} r_{\rm I} A^{1/3}$, where $r_{\rm I}$ is the radius parameter of the imaginary potential, $\beta_\ell^{\rm W}$ is the imaginary deformation parameter and $a_{\rm I}$ is the diffuseness. Implicit in our calculation is

Table 1. Optical potential parameters obtained from the CC analysis, the real strength v_0, the WS parameters (the depth W_0, radius parameter r_I and diffuseness a_I) and the deformation lengths for the 2^+ and 4^+ states δ_2 and δ_4, respectively.

Energy (MeV)	v_0 (MeV)	W_0 (MeV)	r_I (fm)	a_I (fm)	δ_2 (fm)	δ_4 (fm)
		^{6}Li + ^{12}C				
124	75.7	95.0	0.670	1.250	0.32	
169	69.8	47.1	1.531	0.808	1.65	
210	75.7	45.3	1.653	0.762	2.04	
					1.63[a], 1.13[b]	
		^{6}Li + ^{28}Si				
210	58.4	27.4	1.646	0.95	1.75	
					1.48[a], 1.15[b]	
		^{6}Li + ^{58}Ni				
71	66.0	17.0	1.530	0.810	0.95	0.59
210	58.9	29.2	1.640	0.770	0.70	
					0.85[a], 1.17[c]	0.51[d]
		^{7}Li + ^{12}C				
132	63.0	31.5	1.600	0.960	1.72	

([a]) Reference [12] (electromagnetic measurements).
([b]) Reference [14].
([c]) Reference [16].
([d]) Reference [20] (theoretical studies).

the assumption that the deformation parameters β_ℓ apply equally to the real, absorptive and Coulomb portions of the potential, *i.e.* $\beta_\ell^{\rm R} = \beta_\ell^{\rm W} = \beta_\ell^{\rm C}$.

Elastic- and inelastic-scattering calculations were carried out using the CC computer code CHUCK3 [21]. The radial integration was carried out to a maximum radius of 20 fm in steps of 0.05 fm. In the initial CC calculations only β_ℓ was allowed to vary using the best-fits optical potential parameters obtained from the DWBA calculations in ref. [5]. However, the fits to the elastic-scattering data were somewhat worse than the folding model fits of ref. [5]. Since it is well established that if the real potential is unable to reproduce the elastic-scattering data it is impossible to cure or absorb this disability by changing the imaginary part [22], *i.e.* the value of the real strength, v_0 [5] is independent of the chosen set of the imaginary WS parameters. Therefore, the next searches were carried out on the optical potential parameters, v_0 and the WS parameters (W_0, r_I and a_I) as well as the β_ℓ values in order to obtain the best fits to the elastic-scattering data. It should be noted that, once the values of these parameters are chosen to fit the elastic-scattering data, there are no additional adjustable parameters left for the angular distributions of inelastic-scattering cross-sections.

In case of ^{6}Li + ^{58}Ni data at 71 MeV, there are two excited states (2^+ and 4^+). To investigate the effect of the CC mechanism on the elastic-scattering calculations we considered first the coupling between the ground state and the first-excited state (2^+) only. Second, we introduced the coupling between the ground state and both excited states and the intermediate coupling between the 2^+ and 4^+ states (coupling the ground and 4^+ states via the 2^+ one). It was found that the coupling to the 4^+ state has a noticeable effect on both the elastic scattering and the inelastic scattering to the 2^+ state. Results of the second procedure are only shown in the present work.

3 Results and discussion

Elastic and inelastic ^{6}Li + ^{12}C scattering at 124, 169 and 210 MeV are analyzed for the low-lying states, 0^+, 2^+ (4.44 MeV), of ^{12}C using the calculated DF potential. Results of the CC calculation described by the parameters listed in table 1 are shown in fig. 1 in comparison with the experimental data. It is clear from table 1 that the obtained real strengths of the S1Y potential used to describe the elastic-scattering cross-sections through the CC treatment are slightly greater than the corresponding strengths found using the single-channel (DWBA) calculations [5] for the three considered sets. The relative increase in depth increases as energy increases. A similar result was obtained [23] for the folded potentials based on the KH and JLM effective NN interactions. Also, it was found [14] that the real volume integrals per interacting nucleon pair of the unique phenomenological potentials used in the CC calculation are slightly larger than those used in the DWBA method.

From fig. 1 we notice that CC calculations show successful agreements with elastic-scattering data similar to those found before [6] using a single-channel treatment. The predicted inelastic-scattering cross-sections using the

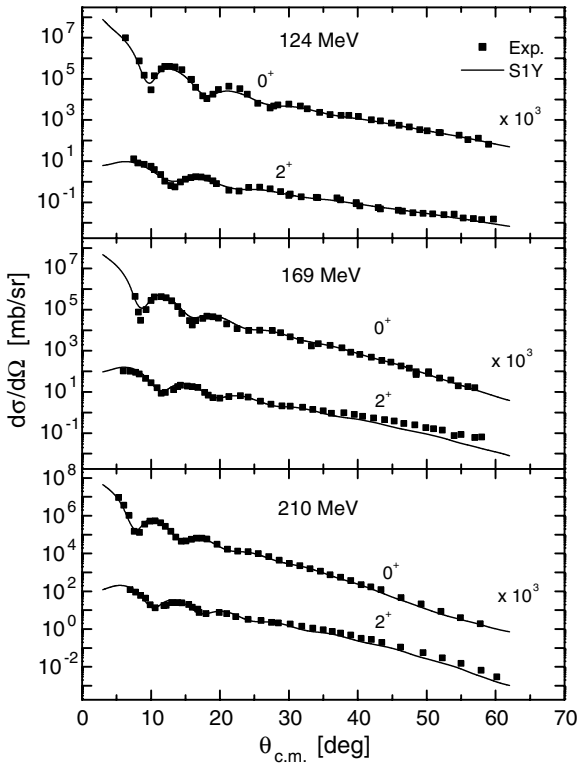

Fig. 1. Angular distributions of elastic and inelastic scattering of ^{6}Li + ^{12}C to the 2^+ (4.44 MeV) state at 124, 169 and 210 MeV. Data are taken from refs. [14,15].

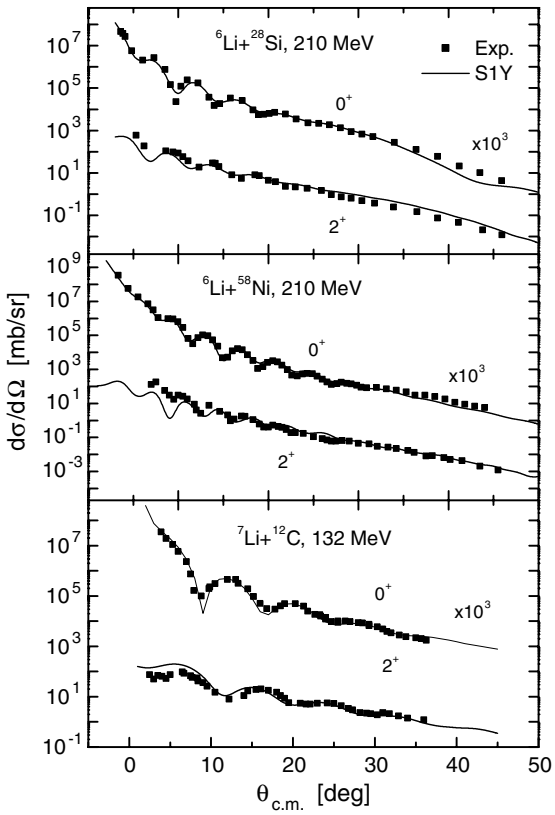

Fig. 2. Angular distributions of elastic and inelastic scattering of ^{6}Li + ^{28}Si to the 2^+ (1.78 MeV) state, ^{6}Li + ^{58}Ni to the 2^+ (1.45 MeV) state at 210 MeV and ^{7}Li + ^{12}C to the 2^+ (4.44 MeV) state at 132 MeV. Data are taken from refs. [14,15].

derived S1Y potential have good agreement with data. The success of the derived spherical and transition potentials to reproduce elastic and inelastic ^{6}Li + ^{12}C scattering cross-sections at 124 and 169 MeV using the CC calculation is better than that obtained by Katori *et al.* [15] using DF optical potentials based on the M3Y interaction through the coupled-discretized-continuum-channels (CDCC) calculations, in which projectile breakup, $^{6}\text{Li} \Rightarrow \alpha + \text{d}$, effects were taken into account. Also, at 210 MeV, the present fits to elastic- and inelastic-scattering data shown in fig. 1 are similar to those resulted from the CC calculations using the unique phenomenological WS optical potential [14].

Nadasen *et al.* [14] and Williamson *et al.* [16] considered the appropriate scaling of the deformation lengths, δ_ℓ, instead of that of the deformation parameters, β_ℓ, as mentioned in the previous section. They used equal deformation lengths for the real, imaginary and Coulomb potentials. It was found [16] that both scalings yielded almost similar results. Therefore, it may be interesting to compare our extracted deformation lengths, δ_2, with those obtained from previous analysis using phenomenological potentials [14] or from electromagnetic measurements [12]. From table 1 we notice that, for ^{6}Li + ^{12}C at 210 MeV, the obtained value of δ_2, 2.04 fm, is more consistent with the observed one [12], 1.63 fm, than those found using the phenomenological unique potential [14], 1.13 and 1.03 fm. The obtained value from the fitting with the 169 MeV data, 1.65 fm, is in excellent agreement with the observed one. However, the extracted value from the 124 MeV is very low compared with the observed one. The previous analysis of the 124 and 169 MeV data [15] did not report, unfortunately, the obtained value of δ_2.

For ^{6}Li + ^{28}Si, angular distributions of the elastic and inelastic scattering are analyzed only at 210 MeV through the CC calculation considering the low-lying states, 0^+, 2^+ (1.78 MeV), of ^{28}Si. Figure 2 illustrates how the derived S1Y potential excellently reproduce the elastic- and inelastic-scattering data. The obtained fits are similar to those resulted from the CC calculation using a phenomenological optical potential [14] and also to those yielded from the CDCC calculation considering the projectile breakup contribution performed by Sakuragi *et al.* [4]. The resulted value of the deformation length, $\delta_2 =$ 1.75 fm, also, shows better agreement with the observed one, 1.48 fm, from electromagnetic measurements [12], than those obtained by the phenomenological potential, 1.15 and 1.21 fm [14] using the DWBA and CC methods, respectively.

The derived DF potential is also used to analyze the elastic and inelastic scattering of ^{6}Li + ^{58}Ni at 71 MeV considering the two excited, 2^+ (1.45 MeV) and 4^+ (2.46 MeV) states of ^{58}Ni. Figure 3 shows reasonable fits with both the elastic and inelastic data. These fits are comparable to those found from the CC analysis using

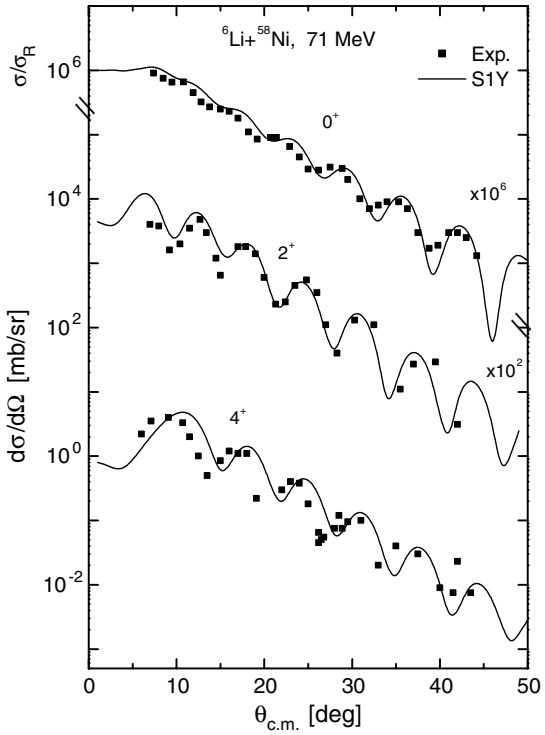

Fig. 3. Angular distributions of elastic and inelastic scattering of ^{6}Li + ^{58}Ni to the 2^+ (1.45 MeV) and 4^+ (2.46 MeV) states at 71 MeV. Data are taken from ref. [16].

different sets of phenomenological potentials [16] and also comparable to those obtained from the DWBA analysis using the M3Y DF potentials [24]. The present fits are better than those obtained using folded potentials based on the KH and JLM effective NN interactions [23], particularly at backward angles. The obtained deformation lengths at 71 MeV are quite consistent with those reported previously by Williamson *et al.* [16] and the references therein. The extracted value of δ_2 is 0.95 fm, while the average value obtained from several previous studies [16] is 0.99 ± 0.12 fm. The deformation length $\delta_4 = 0.59$ fm is in good agreement with the reported average value from previous studies, 0.51 ± 0.073 fm.

For ^{6}Li + ^{58}Ni at 210 MeV the strength of the S1Y potential is slightly greater than the corresponding one using the single-channel calculations [5]. The present fits with data, shown in fig. 2, are very similar to those obtained from the CC analysis performed using the unique phenomenological potential [14] for both elastic and inelastic results. As shown in table 1, the obtained deformation length, $\delta_2 = 0.70$ fm, is also consistent with both the observed value, 0.85 fm [12], and those resulted from DWBA and CC analyses using phenomenological potentials [14], 0.76 and 0.8 fm, respectively.

The CC calculations are performed for ^{7}Li + ^{12}C scattering data only at 132 MeV. Figure 2 shows excellent agreements between the elastic and inelastic data and our results. These agreements are clearly better than those obtained by Katori *et al.* [15] from CC calculations using the M3Y DF potential, particularly, for the inelastic scattering. The fits obtained using the CC calculation are slightly better than that obtained by the DWBA calculations [5]. The obtained quadrupole deformation length agrees well with the measured value.

4 Conclusions

The ^{6,7}Li nucleus DF optical potentials are generated based on the S1Y effective NN interaction. Eight sets of ^{6,7}Li inelastic-scattering data are analyzed using the derived potentials in the framework of the CC mechanism. Excellent reproductions of the inelastic and the corresponding elastic data are obtained. In the mean time, it is observable that the consistency between the extracted deformation lengths with the measured values is better than that found from previous analyses using phenomenological optical potentials.

On the other hand, we conclude that the CC calculations revealed an insignificant improvement in the fits with data compared with those obtained using the DWBA calculation. However, for ^{6}Li + ^{58}Ni reaction at 71 MeV, the coupling to the second-excited state (4^+) besides the first one (2^+) substantially improved the fit with the elastic scattering data. In addition, the strengths of the S1Y interaction required for the CC calculations are, in general, slightly larger than those used in the DWBA method.

Finally, from our previous [5,6] and present studies, we confirm that the S1Y effective NN interaction has a powerful ability to reproduce successful descriptions of ^{6,7}Li elastic- and inelastic-scattering data at intermediate energies.

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